Chapter 4

1

Motion in two dimensions

1

Lecture 03

2

Circular Motion

2

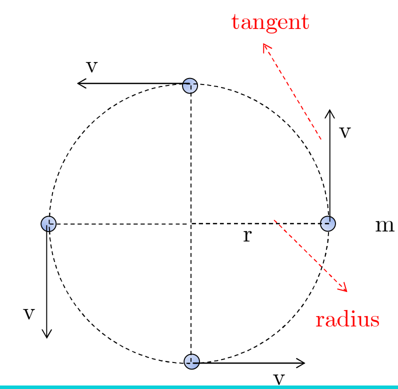
Uniform Circular Motion

3

Uniform circular motion occurs when an object moves in a circular path with a constant speed.

The constant-magnitude velocity vector is always tangent to the path of the object.

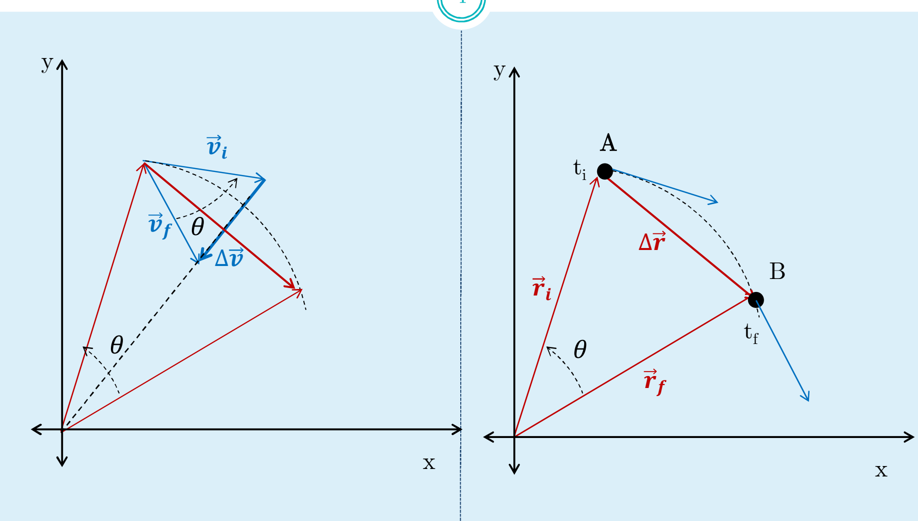
An acceleration exists since the direction of the motion is changing .



3

Changing Velocity in Uniform Circular Motion

4



4

Changing Velocity in Uniform Circular Motion

5

The change in the velocity vector is due to the change in direction.

The direction of the change in velocity is toward the center of the circle.

The vector diagram shows:

- $\vec{v}_f = \vec{v}_i + \Delta\vec{v}$

5

Centripetal Acceleration

6

The acceleration is always perpendicular to the path of the motion.

The acceleration always points toward the center of the circle of motion.

This acceleration is called the centripetal acceleration.

- It is also called the radial acceleration.

6

Centripetal Acceleration, cont

7

The magnitude of the centripetal acceleration vector is given by

- $a_c = \frac{v^2}{r}$

The direction of the centripetal acceleration vector is always changing, to stay directed toward the center of the circle of motion.

7

Period

8

The period, T, is the time required for one complete revolution.

The speed of the particle would be the circumference of the circle of motion divided by the period.

- $v = \frac{2\pi r}{T}$

Therefore, the period is defined as

- $T = \frac{2\pi}{v}$

8

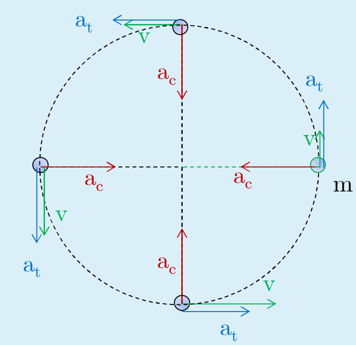
Tangential Acceleration

9

The magnitude of the velocity could also be changing.

In this case, there would be a tangential acceleration.

The motion would be under the influence of both tangential and centripetal accelerations.



9

Tangential Acceleration, equation

10

The tangential acceleration:

- $a_t = \left|\frac{dv}{dt}\right|$
- Direction: Same as velocity vector if (v) is increasing, opposite if (v) is decreasing

10

Total Acceleration

11

The tangential acceleration causes the change in the speed of the particle.

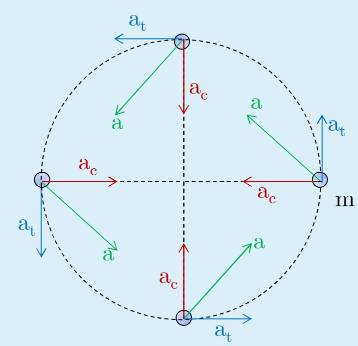
The centripetal acceleration comes from a change in the direction of the velocity vector.

The total acceleration:

$$\vec{a} = \vec{a}_c + \vec{a}_t$$

Its magnitude:

$$a = \sqrt{a_c^2 + a_t^2}$$



11

The Centripetal Acceleration of the Earth

Saturday, 30 January, 2021 12:28

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

SOLUTION

The period of the Earth's orbit, which we know is one year, and the radius of the Earth's orbit around the Sun, which is $1.496 \times 10^{11} m$.

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$a_c = \frac{4\pi^2(1.496 \times 10^{11} m)}{(1yr)^2}\left(\frac{1yr}{3.156 \times 10^7 s}\right)^2 = 5.93 \times 10^{-3} m/s^2$$

The Centripetal Acceleration

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An object moves at constant speed along a circular path in a horizontal xy plane, with the center at the origin .When the object is at $x = -2\ m$, its velocity is $(-4\ m/s)\ \hat{j}$. Give the object's velocity and acceleration at $y = 2\ m$.

Solution

$$\vec{v} = -(4\ \mathrm{m}/s)\hat{\imath}$$

$$\vec{a}_c = \frac{v^2}{r}(-\hat{\jmath}) = \frac{4^2}{2}(-\hat{\jmath}) = -8(\hat{\jmath})m/s^2$$

Over the Rise

Saturday, 30 January, 2021 12:29

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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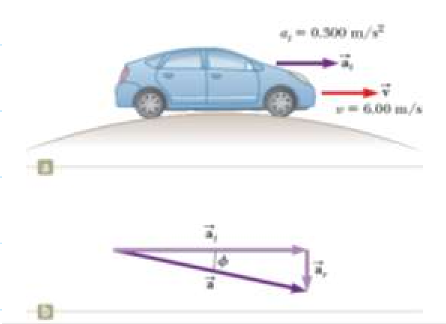
A car leaves a stop sign and exhibits a constant acceleration of $0.3\ m/s^2$ parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius $500\,m$. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of $6\ m/s$. What are the magnitude and direction of the total acceleration vector for the car at this instant?



SOLUTION

The tangential acceleration vector has magnitude ($0.3\ m/s^2$) and is horizontal.

the radial acceleration:

$$a_r = -\frac{v^2}{r} = -\frac{(6.00 m/s)^2}{500 m} = -0.0720 m/s^2$$

Find the magnitude of $\vec{a}$:

$$\sqrt{a_r^2 + a_t^2} = \sqrt{(-0.0720 m/s^2)^2 + (0.300 m/s^2)^2} = 0.309 m/s^2$$

Find the angle ϕ between $\vec{a}$ and the horizontal:

$$\phi = tan^{-1}\frac{a_r}{a_t} = tan^{-1}\left(\frac{-0.0720 m/s^2}{0.300 m/s^2}\right) = -13.5°$$